

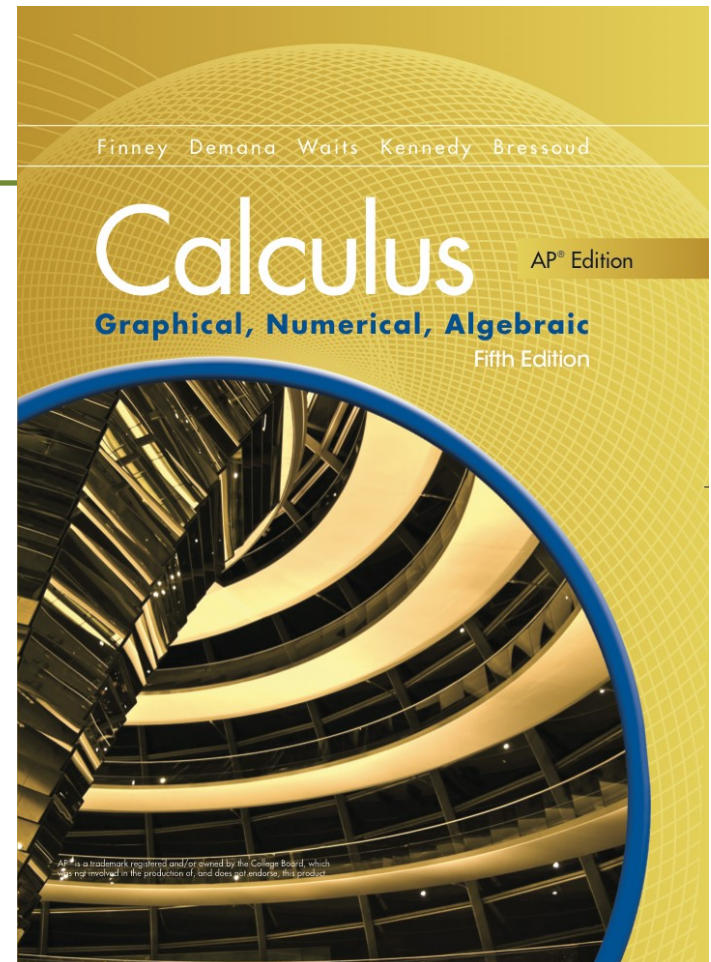
Chapter 5

Applications of Derivatives



Section 5.1

Extreme Values of Functions



Quick Review

Find the first derivative of the function.

1. $f(x) = \sqrt{3-x}$

2. $f(x) = \frac{1}{\sqrt{x^2 - 1}}$

Find the limits for $f(x) = \frac{3}{\sqrt{4-x^2}}$.

3. $\lim_{x \rightarrow 2^+} f(x)$

4. $\lim_{x \rightarrow 2^-} f(x)$

Quick Review

$$\text{Let } f(x) = \begin{cases} x^2 + 1, & x \leq 2 \\ x - 4, & x > 2 \end{cases}$$

5. Find $f'(1)$.
6. Find $f'(2)$.
7. Find the domain for $f'(x)$.
8. Write a formula for $f'(x)$.

Quick Review Solutions

Find the first derivative of the function.

$$1. f(x) = \sqrt{3-x} \quad f'(x) = \frac{-1}{2\sqrt{3-x}}$$

$$2. f(x) = \frac{1}{\sqrt{x^2-1}} \quad f'(x) = \frac{-x}{(x^2-1)^{3/2}}$$

Find the limits for $f(x) = \frac{3}{\sqrt{4-x^2}}$.

$$3. \lim_{x \rightarrow 2^+} f(x) \text{ does not exist}$$

$$4. \lim_{x \rightarrow 2^-} f(x) = \infty$$

Quick Review Solutions

$$\text{Let } f(x) = \begin{cases} x^2 + 1, & x \leq 2 \\ x - 4, & x > 2 \end{cases}$$

5. Find $f'(1)$. 2

6. Find $f'(2)$. Undefined

7. Find the domain for $f'(x)$. $x \neq 2$

8. Write a formula for $f'(x)$. $f'(x) = \begin{cases} 2x, & x < 2 \\ 1, & x > 2 \end{cases}$

What you'll learn about

- Absolute (Global) and Local (Relative) Extrema
- The Extreme Value Theorem
- Using the derivative to find extrema

...and why

Finding maximum and minimum values of a function, called optimization, is an important issue in real-world problems.

Absolute Extreme Values

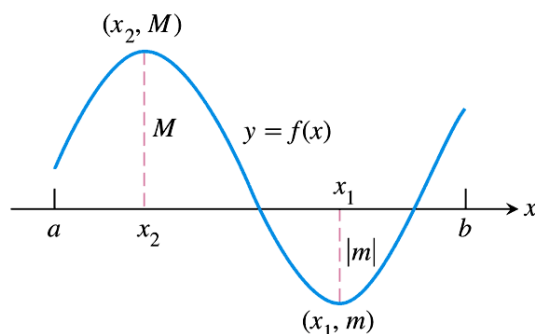
Let f be a function with domain D . Then $f(c)$ is the

(a) absolute maximum value on D if and only if $f(x) \leq f(c)$ for all x in D .

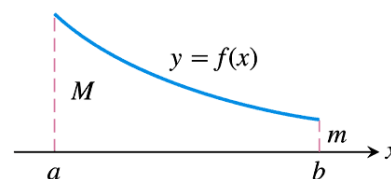
(b) absolute minimum value on D if and only if $f(x) \geq f(c)$ for all x in D .

The Extreme Value Theorem

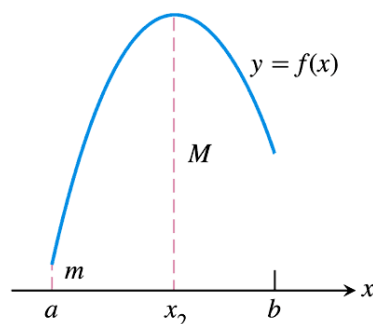
If f is continuous on a closed interval $[a, b]$, then f has both a maximum value and a minimum value on the interval.



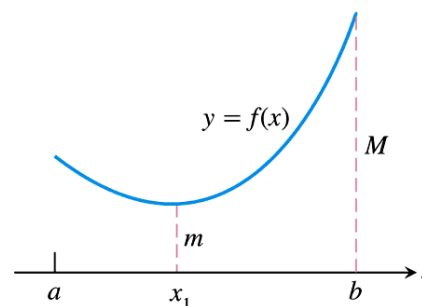
Maximum and minimum
at interior points



Maximum and minimum
at endpoints

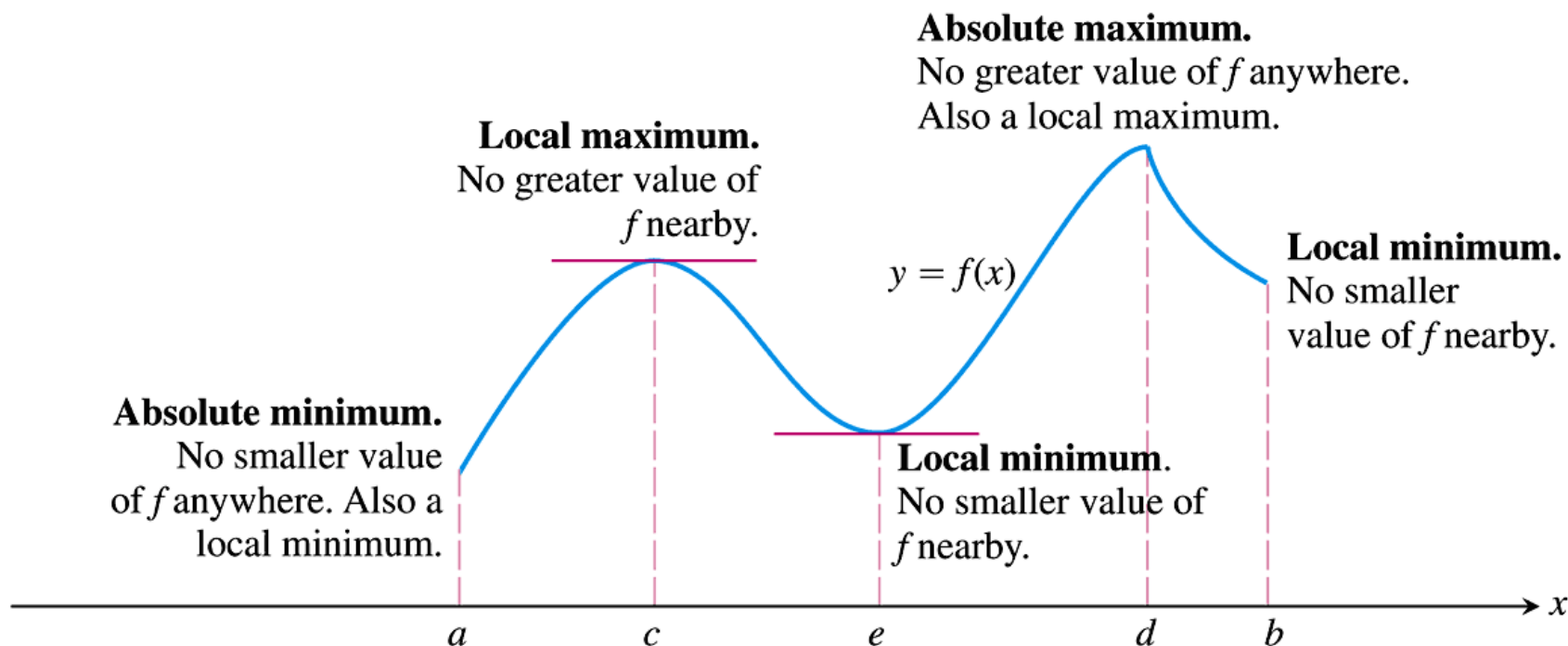


Maximum at interior point,
minimum at endpoint



Minimum at interior point,
maximum at endpoint

Classifying Extreme Values



Local Extreme Values

Let c be an interior point of the domain of the function f . Then $f(c)$ is a
(a) local maximum value at c if and only if $f(x) \leq f(c)$ for all x in some open interval containing c .

(b) local minimum value at c if and only if $f(x) \geq f(c)$ for all x in some open interval containing c .

A function f has a local maximum or local minimum at an endpoint c if the appropriate inequality holds for all x in some half-open domain interval containing c .

Local Extreme Values

If a function f has a local maximum value or a local minimum value at an interior point c of its domain, and if f' exists at c , then $f'(c) = 0$.

Critical Point

A point in the interior of the domain of a function f at which $f' = 0$ or f' does not exist is a **critical point** of f .

Stationary Point

A point in the interior of the domain of a function f at which $f' = 0$ is called a **stationary point** of f .

Example Finding Absolute Extrema

Find the absolute maximum and minimum values of $f(x) = 3x^{2/3}$ on the interval $[-1, 2]$.

Find the critical point values: $f'(x) = \frac{2}{\sqrt[3]{x}}$ has no zeros but is undefined at $x = 0$.

Critical point value: $f(0) = 0$

Endpoint values: $f(-1) = 3$;

$$f(2) = 3(2)^{2/3} \approx 4.762$$

The absolute maximum value is 4.762 and occurs at $x = 2$.

The absolute minimum value is 0 and occurs at $x = 0$.

Example Finding Extreme Values

Find the extreme values of $f(x) = \frac{1}{\sqrt{9-x^2}}$.

f is defined for $9-x^2 > 0$, so its domain is the interval $(-3, 3)$. Since the domain has no endpoints, all the extreme values must occur at critical points.

$$f(x) = \frac{1}{\sqrt{9-x^2}} = (9-x^2)^{-1/2}$$

$$f'(x) = -\frac{1}{2}(9-x^2)^{-3/2}(-2x) = \frac{x}{(9-x^2)^{3/2}}$$

The only critical point is at $x=0$. The only candidate for an extreme value is $f(0) = 1/3$. To determine whether $1/3$ is an extreme value of f , examine $f(x)$. As x moves away from 0 on either side, the denominator gets smaller, the values of f increase and the graph rises. There is an absolute minimum at $x=0$. The function has no maxima, either local or absolute.